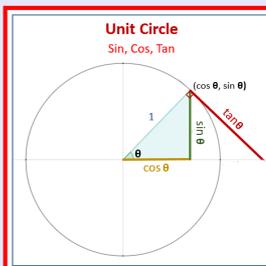


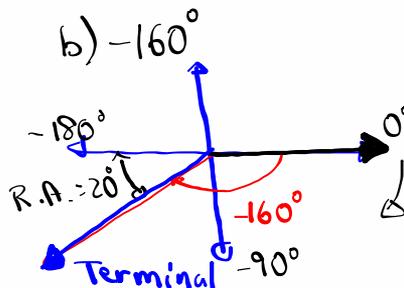
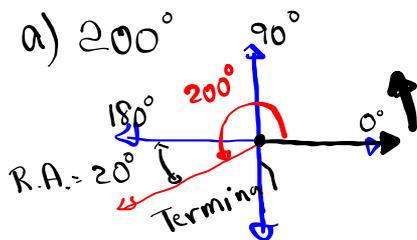
**Math 241**  
**Winter 2024**  
**Lecture 7**



Feb 19-8:47 AM

Class QZ 6 (Bonus)

Draw angles in standard position, find its ref. angle



$200^\circ$  &  $-160^\circ$  are Coterminal angles.

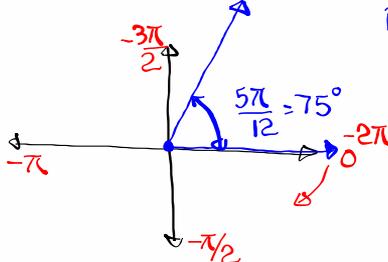
Jan 10-12:05 PM

Convert  $\frac{5\pi}{12}$  to degrees.  $\frac{5\pi}{12}$  Radians =  $\frac{5\pi}{12} \cdot \frac{180}{\pi}$  degrees

Draw in standard position

Give its ref. angle.

Find a negative angle that is Coterminal with  $\frac{5\pi}{12}$ .



Ref. angle  $75^\circ$

$$-2\pi + \frac{5\pi}{12}$$

$$= \frac{-2\pi(12) + 5\pi}{12}$$

$$= \frac{-24\pi + 5\pi}{12}$$

$$= \frac{-19\pi}{12}$$

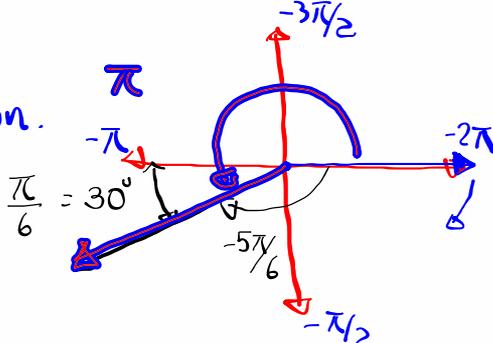
what about in degrees?

$$-360^\circ + 75^\circ = -285^\circ$$

Jan 11-8:07 AM

Given  $\theta = -\frac{5\pi}{6}$

- 1) Draw in Standard Position.
- 2) find its ref. angle.
- 3) Convert to degrees.
- 4) find a positive angle that is Coterminal with  $\theta$ .

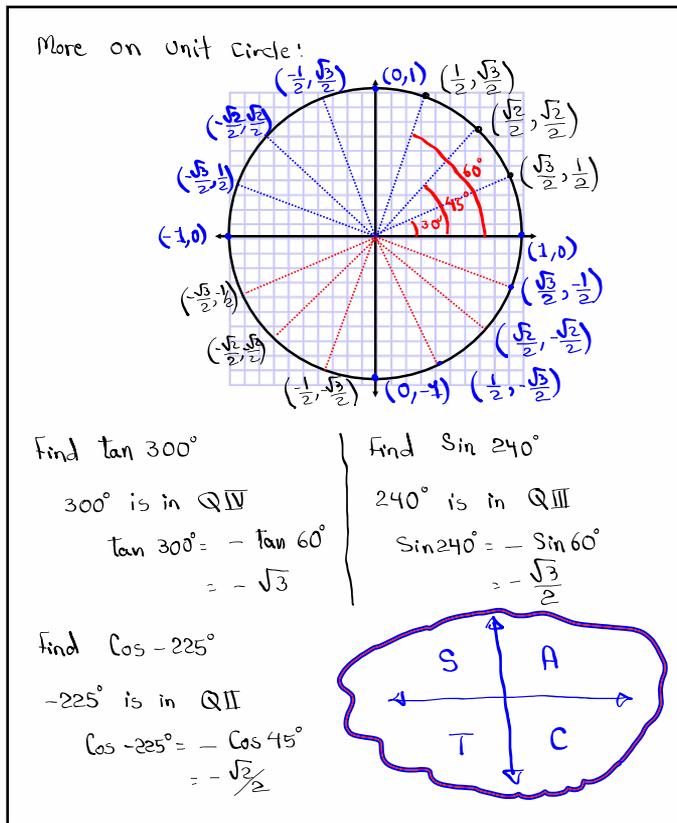


$$-\frac{5\pi}{6} = -5(30^\circ) = -150^\circ$$

$$\pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$180^\circ + 30^\circ = 210^\circ$$

Jan 11-8:16 AM



Jan 11-8:25 AM

Verify

$$\frac{1 + \cot^2 \theta}{\csc^2 \theta - 1} = \sec^2 \theta$$

LHS =  $\frac{1 + \frac{\cos^2 \theta}{\sin^2 \theta}}{\frac{1}{\sin^2 \theta} - 1} = \frac{\sin^2 \theta (1 + \frac{\cos^2 \theta}{\sin^2 \theta})}{\sin^2 \theta (\frac{1}{\sin^2 \theta} - 1)}$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{1 - \sin^2 \theta} = \frac{1}{\cos^2 \theta} = \left(\frac{1}{\cos \theta}\right)^2 = \boxed{\sec^2 \theta} \text{ RHS}$$

Jan 11-8:38 AM

Verify by **Cross-Multiplication**

$$\frac{\cos \alpha}{1 - \sin \alpha} = \frac{1 + \sin \alpha}{\cos \alpha}$$

$$\cos \alpha \cdot \cos \alpha = (1 - \sin \alpha) \cdot (1 + \sin \alpha)$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha$$

$$\cos^2 \alpha = \cos^2 \alpha \checkmark$$

Verify  $\frac{\cos \alpha}{1 - \sin \alpha} = \frac{1 + \sin \alpha}{\cos \alpha}$  by multiplying top & bottom of LHS with the **conjugate** of the **denominator**.

$$\frac{\cos \alpha}{1 - \sin \alpha} \cdot \frac{1 + \sin \alpha}{1 + \sin \alpha} = \frac{\cos \alpha (1 + \sin \alpha)}{(1 - \sin \alpha)(1 + \sin \alpha)}$$

$$= \frac{\cos \alpha (1 + \sin \alpha)}{1 - \sin^2 \alpha}$$

$$= \frac{\cos \alpha (1 + \sin \alpha)}{\cos^2 \alpha} = \frac{1 + \sin \alpha}{\cos \alpha}$$

RHS

Jan 11-8:42 AM

Verify

$$\frac{\sec \alpha + \tan \alpha}{\sec \alpha - \tan \alpha} = \frac{1 + 2 \sin \alpha + \sin^2 \alpha}{\cos^2 \alpha}$$

$$\text{LHS} = \frac{\cos \alpha \left( \frac{1}{\cos \alpha} + \frac{\sin \alpha}{\cos \alpha} \right)}{\cos \alpha \left( \frac{1}{\cos \alpha} - \frac{\sin \alpha}{\cos \alpha} \right)}$$

LCD =  $\cos \alpha$

$$= \frac{1 + \sin \alpha}{1 - \sin \alpha} \cdot \frac{1 + \sin \alpha}{1 + \sin \alpha}$$

$$= \frac{(1 + \sin \alpha)(1 + \sin \alpha)}{(1 - \sin \alpha)(1 + \sin \alpha)}$$

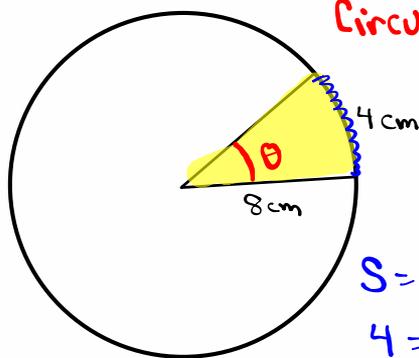
$$= \frac{1 + \sin \alpha + \sin \alpha + \sin^2 \alpha}{1 - \sin^2 \alpha}$$

$$= \frac{1 + 2 \sin \alpha + \sin^2 \alpha}{\cos^2 \alpha}$$

RHS

Jan 11-8:50 AM

Find the shaded area below



Circular Sector

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \cdot 8^2 \cdot \frac{1}{2}$$

$$s = r \theta$$

$$4 = 8 \cdot \theta$$

$$\theta = \frac{1}{2}$$

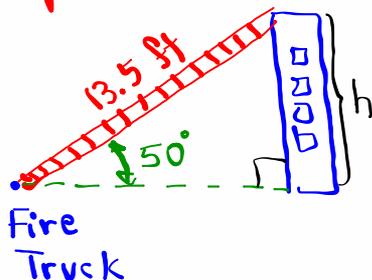
$$= \frac{64}{4} = \boxed{16 \text{ cm}^2}$$

Jan 11-8:58 AM

A fire-truck uses a 13.5 ft ladder to reach the top of a building.

The angle of elevation from fire truck to the top of the building is  $50^\circ$ .

Find how tall the building is. **Complete drawing required.**



$$\sin 50^\circ = \frac{h}{13.5}$$

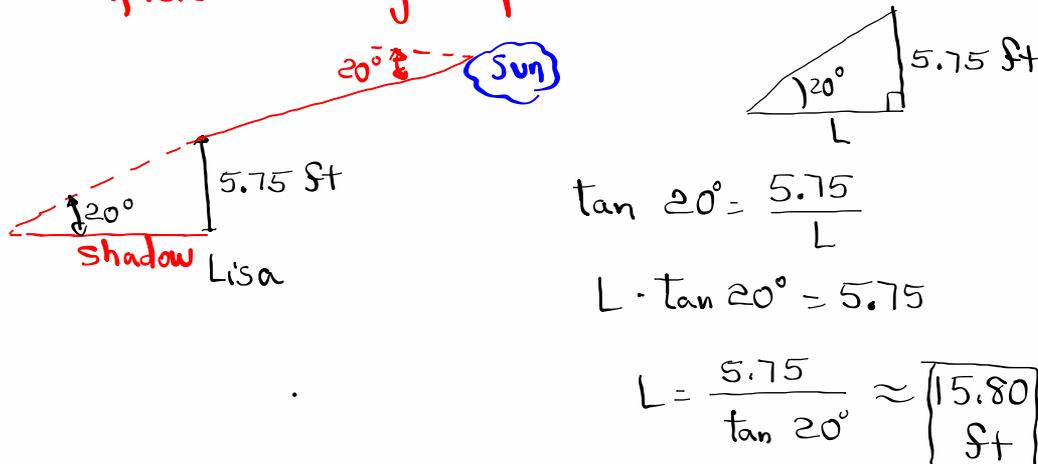
$$h = 13.5 \sin 50^\circ$$

$$\approx \boxed{10.3 \text{ ft}}$$

Jan 11-9:03 AM

The angle of depression from the sun to the head of Lisa who is 5.75 ft tall is  $20^\circ$ .  
Find the length of her shadow.

Complete drawing required.



$$\tan 20^\circ = \frac{5.75}{L}$$

$$L \cdot \tan 20^\circ = 5.75$$

$$L = \frac{5.75}{\tan 20^\circ} \approx \boxed{15.80 \text{ ft}}$$

Jan 11-9:10 AM

Verify

$$\tan^2 \alpha \cdot \sin^2 \alpha + 1 = \tan^2 \alpha + \cos^2 \alpha$$

$$\text{LHS} = \tan^2 \alpha (1 - \cos^2 \alpha) + 1$$

$$= \tan^2 \alpha - \tan^2 \alpha \cdot \cos^2 \alpha + 1$$

$$= \tan^2 \alpha - \frac{\sin^2 \alpha}{\cos^2 \alpha} \cdot \cos^2 \alpha + 1$$

$$= \tan^2 \alpha - \sin^2 \alpha + 1$$

$$= \tan^2 \alpha + \underbrace{1 - \sin^2 \alpha}$$

$$= \boxed{\tan^2 \alpha + \cos^2 \alpha} \text{ RHS}$$

Jan 11-9:34 AM

Verify

$$(1 + \sin x + \cos x)^2 = 2(1 + \sin x)(1 + \cos x)$$

$$\text{LHS} = (1 + \sin x + \cos x)(1 + \sin x + \cos x)$$

$$= 1 + \sin x + \cos x + \sin x + \sin^2 x + \sin x \cos x$$

$$+ \cos x + \sin x \cos x + \cos^2 x$$

$$= 2 + 2\sin x + 2\cos x + 2\sin x \cos x$$

$$= 2(1 + \sin x + \cos x + \sin x \cos x)$$

$$= 2 \left[ \underbrace{1 + \sin x} + \cos x \underbrace{(1 + \sin x)} \right]$$

$$= 2 \left[ (1 + \sin x)(1 + \cos x) \right]$$

RHS

Jan 11-9:39 AM

Simplify

$$(2\sin x + \cos x)^2 + (2\cos x - \sin x)^2$$

$$= 4\sin^2 x + 4\cancel{\sin x \cos x} + \cos^2 x + 4\cos^2 x - 4\cancel{\sin x \cos x} + \sin^2 x$$

$$= 5\sin^2 x + 5\cos^2 x$$

$$= 5(\sin^2 x + \cos^2 x) = 5 \cdot 1 = \boxed{5}$$

Jan 11-9:47 AM

Verify

$$\sec x - \cos x + \csc x - \sin x - \sin x \tan x = \boxed{\cos x \cot x}$$

$$\text{LHS} = \frac{1}{\cos x} - \cos x + \frac{1}{\sin x} - \sin x - \frac{\sin x \cdot \sin x}{\cos x}$$

$\text{LCD} = \cos x \cdot \sin x$

$$= \frac{\sin x - \cos x \cdot \cos x \cdot \sin x + \cos x - \sin x \cdot \cos x \sin x - \sin^3 x}{\cos x \cdot \sin x}$$

$$= \frac{\sin x - \sin x \cos^2 x + \cos x - \sin^2 x \cos x - \sin^3 x}{\sin x \cos x}$$

$$= \frac{\sin x (1 - \cos^2 x) + \cos x - \sin^2 x \cos x - \sin^3 x}{\sin x \cos x}$$

$$= \frac{\cancel{\sin x} \cdot \sin^2 x + \cos x - \sin^2 x \cos x - \cancel{\sin^3 x}}{\sin x \cos x}$$

$$= \frac{\cancel{\cos x} (1 - \sin^2 x)}{\cancel{\sin x} \cos x} = \frac{\cos^2 x}{\sin x} = \frac{\cos x}{\sin x} \cdot \cos x$$

$$= \boxed{\cot x \cdot \cos x}$$

RHS

Jan 11-9:54 AM

A plane flies 1.3 hrs at 110 mph on a bearing of  $38^\circ$ .

$1.3(110) = 143$  miles

Then it turns and flies 1.5 hrs on a bearing of  $128^\circ$ .

$1.5(110) = 165$  miles

How far is the plane from the starting point?

By Pythagorean thrm

$$d^2 = 143^2 + 165^2$$

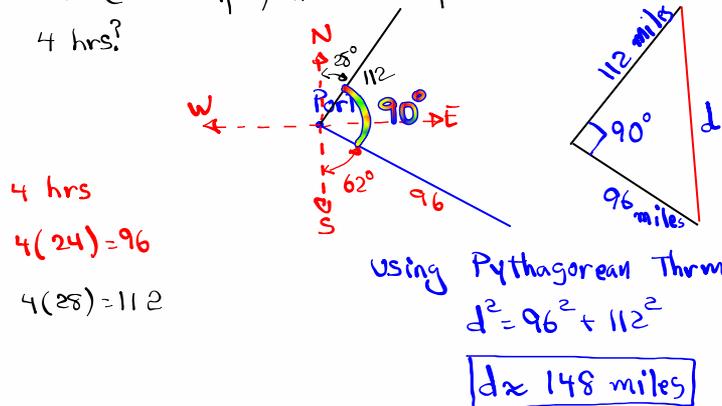
$$d \approx \boxed{218 \text{ miles}}$$

Jan 11-10:08 AM

A ship leaves its home port on a bearing of S  $62^\circ$  E.

Another ship leaves the same port on a bearing of N  $28^\circ$  E.

If the first ship sails @ 24 mph and second ship sails @ 28 mph, how far apart are they after 4 hrs?



Jan 11-10:17 AM

Bearing:

Method I:

Measure from North axis, clockwise

Method II: Measure from North and South in the direction of East or West

N  $10^\circ$  E , N  $70^\circ$  W

S  $15^\circ$  E , S  $10^\circ$  W

Jan 11-10:26 AM

Bearing from A to B is  $S 82^\circ W$ .

Bearing from A to C is  $N 64^\circ W$ .

Bearing from B to C is  $N 26^\circ E$ .

A plane is flying @ 350 mph takes 1.8 hrs to fly from A to B.  $350(1.8) = 630$  miles

Find the distance from B to C.

$\sin 34^\circ = \frac{x}{630}$   
 $x = 630 \cdot \sin 34^\circ$   
 $x \approx 352$  miles

Jan 11-10:28 AM

Intro. to graphing:

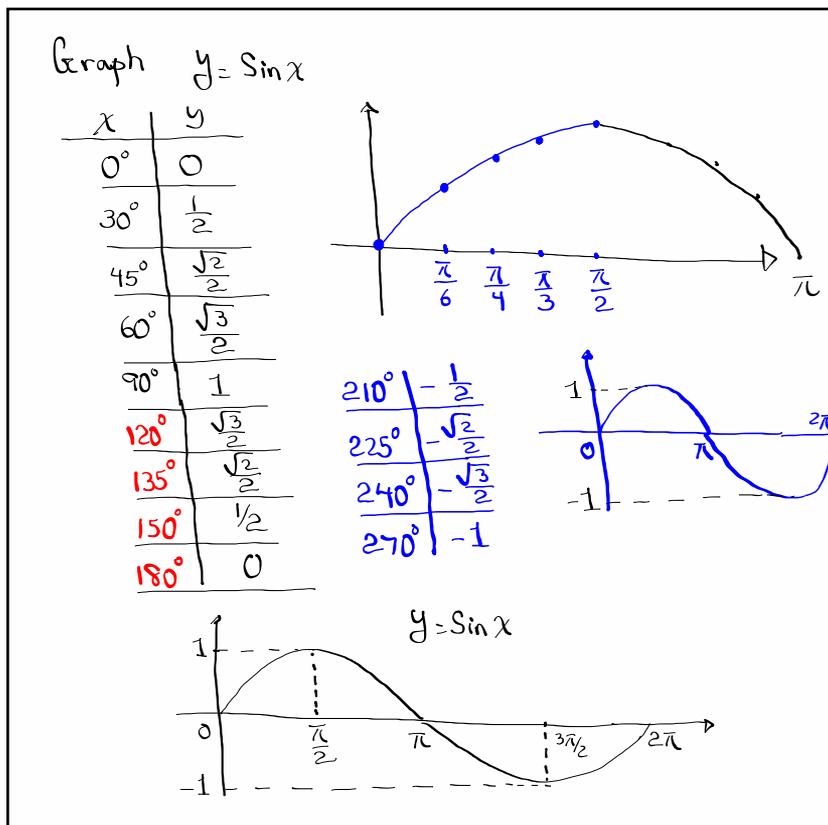
Graph  $y = x^2$

x	y
0	0
1	1
-1	1
2	4
-2	4

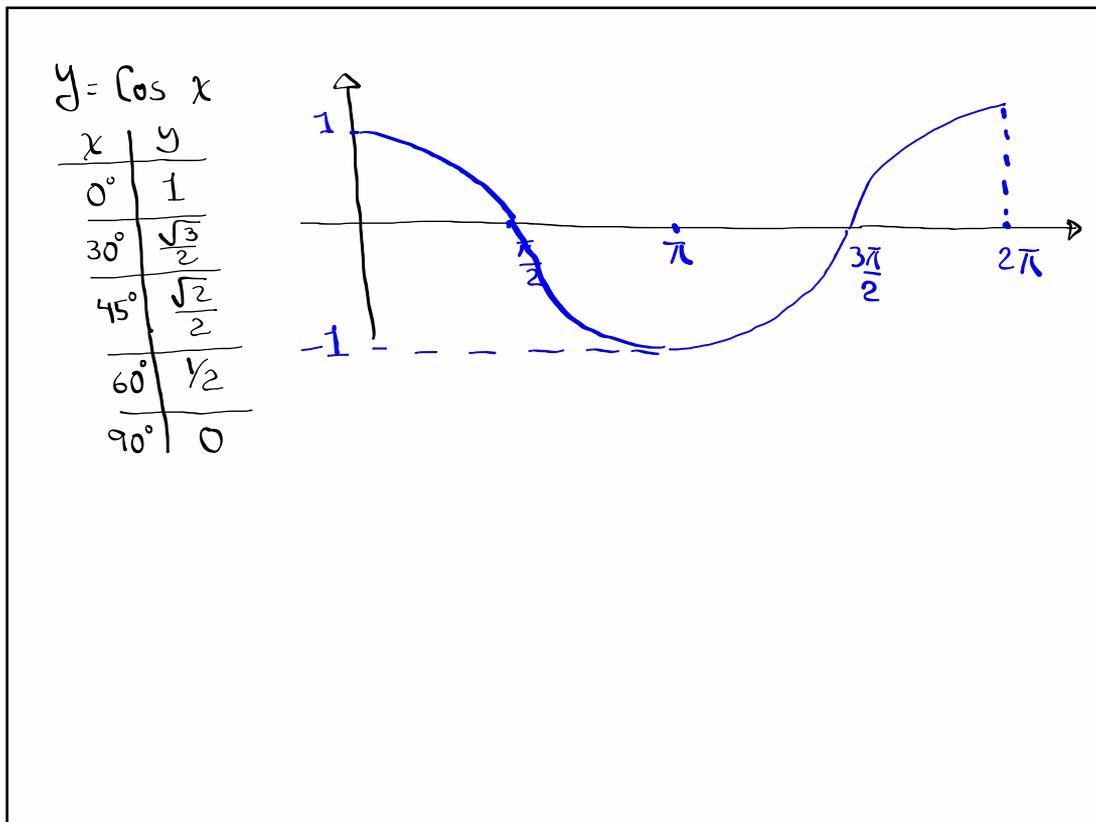
$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

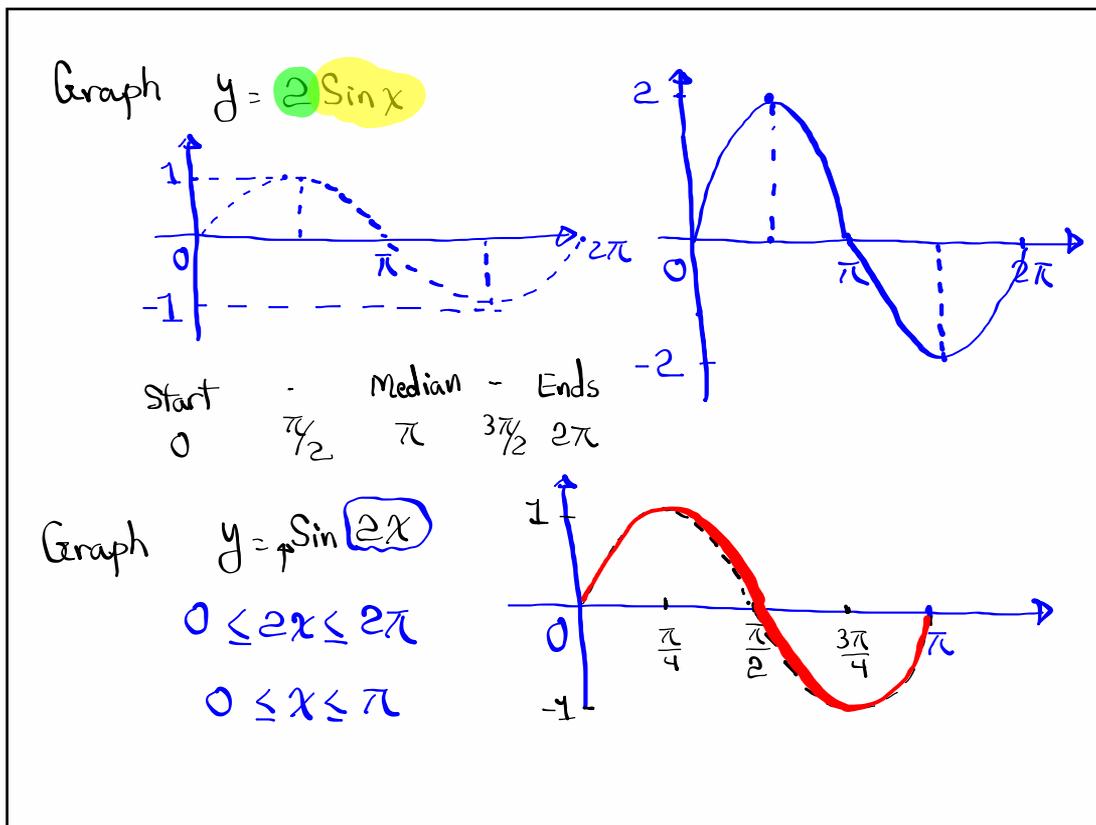
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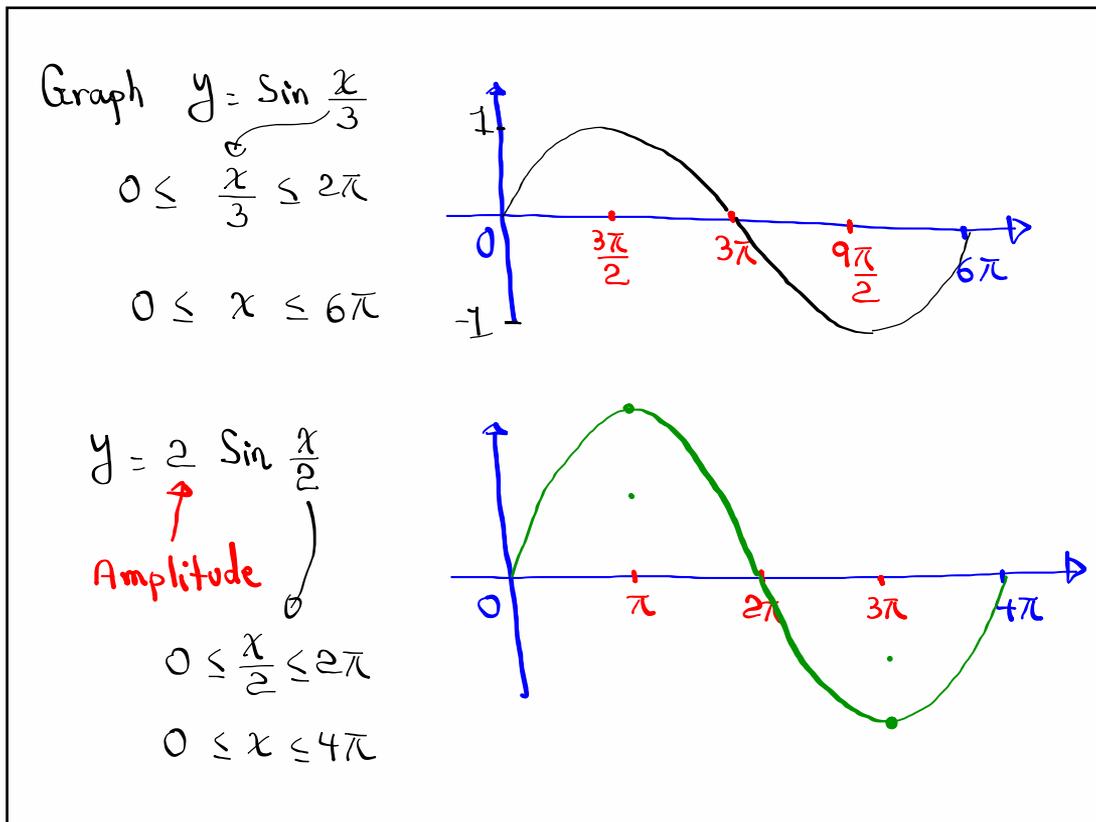
Jan 11-11:17 AM



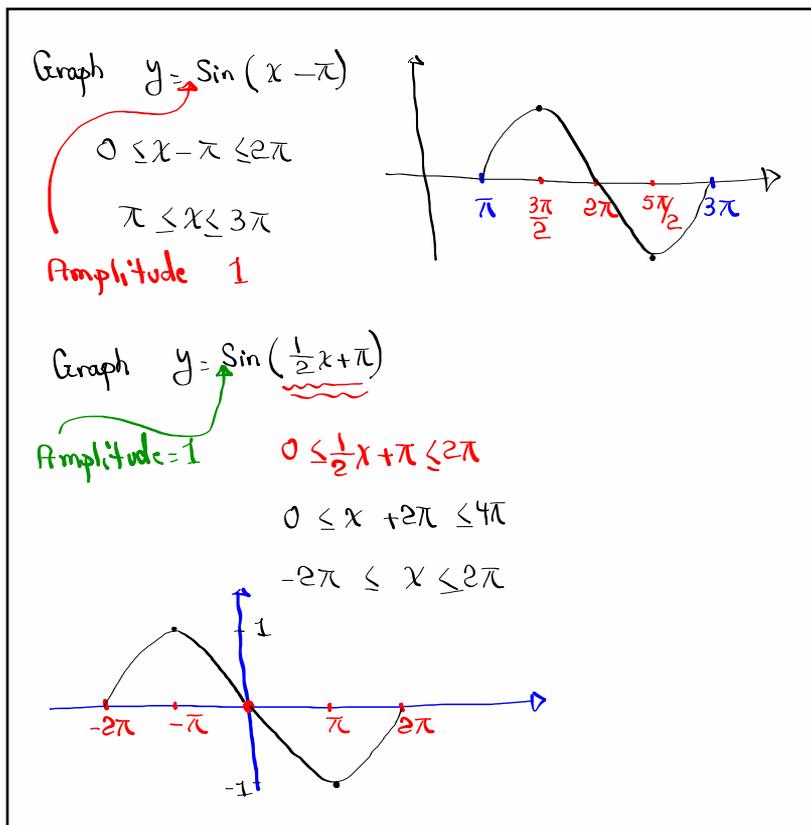
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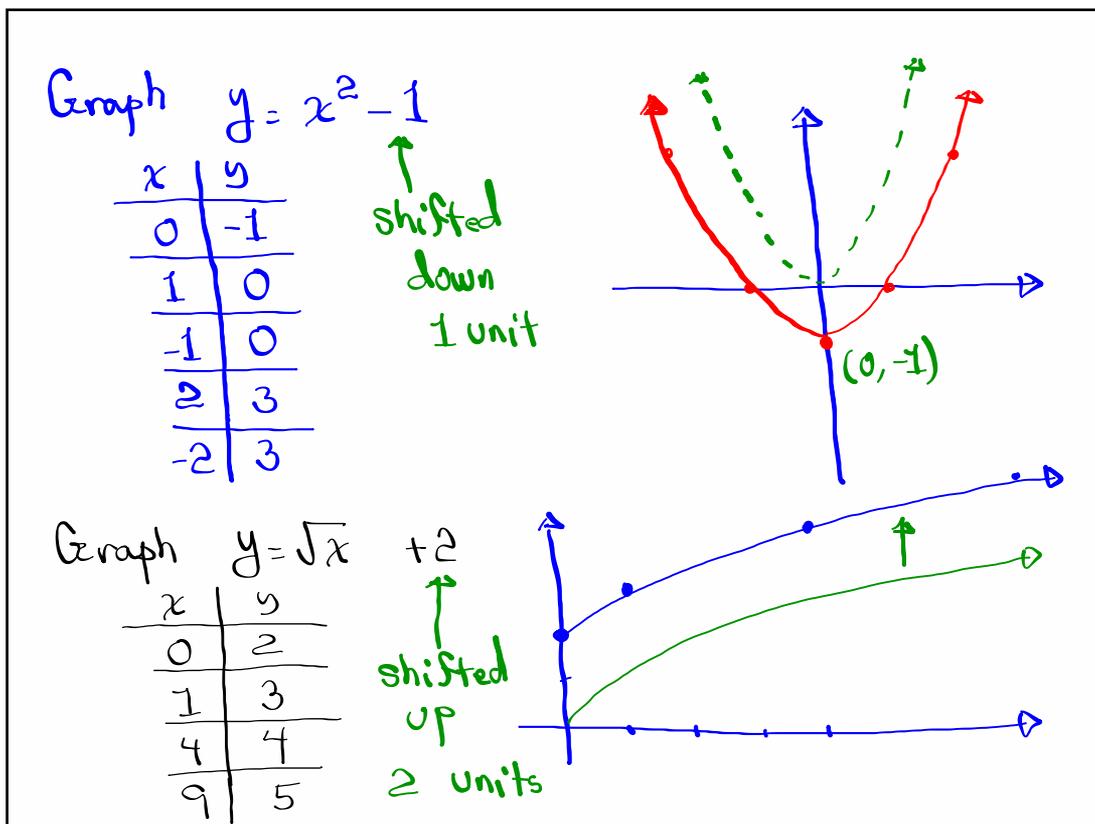
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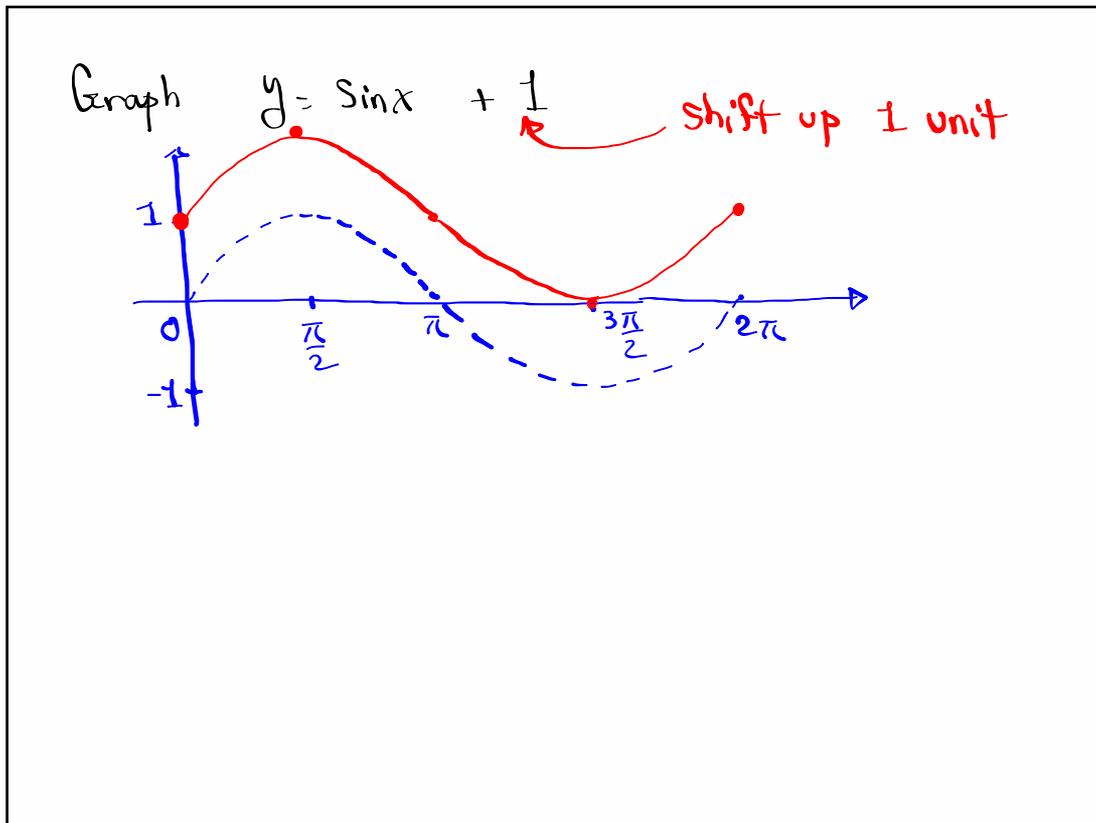
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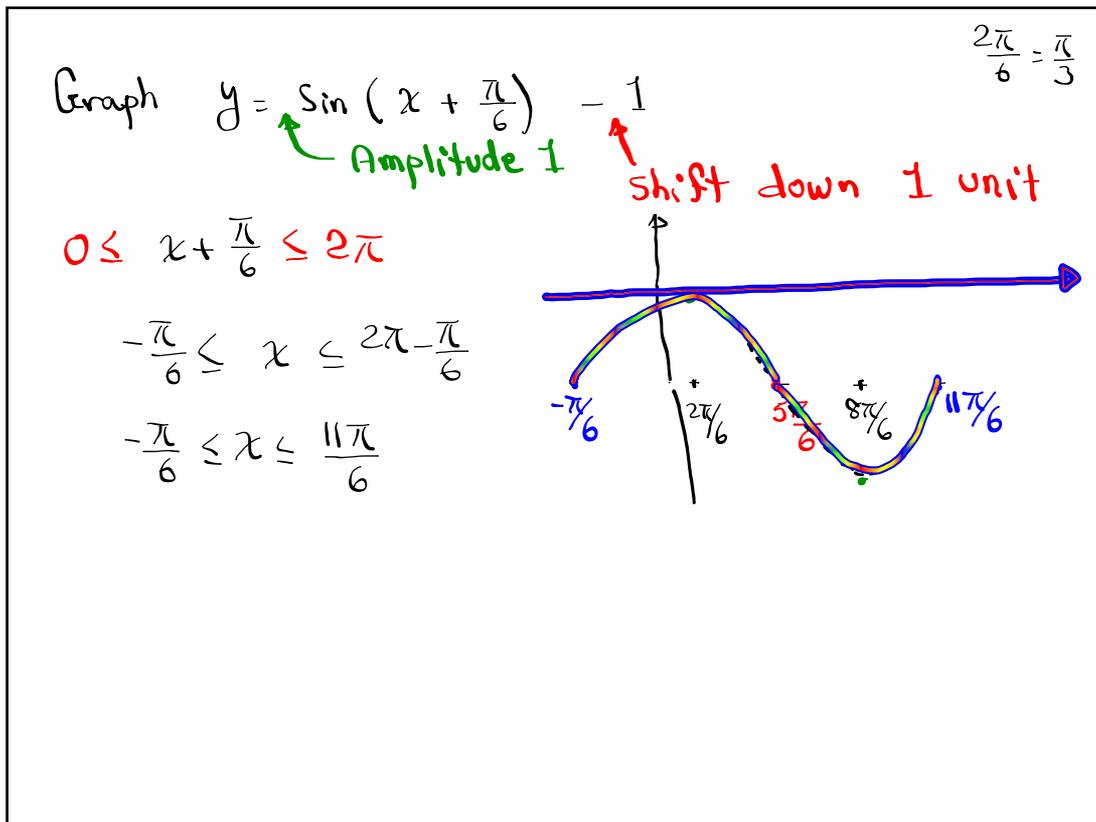
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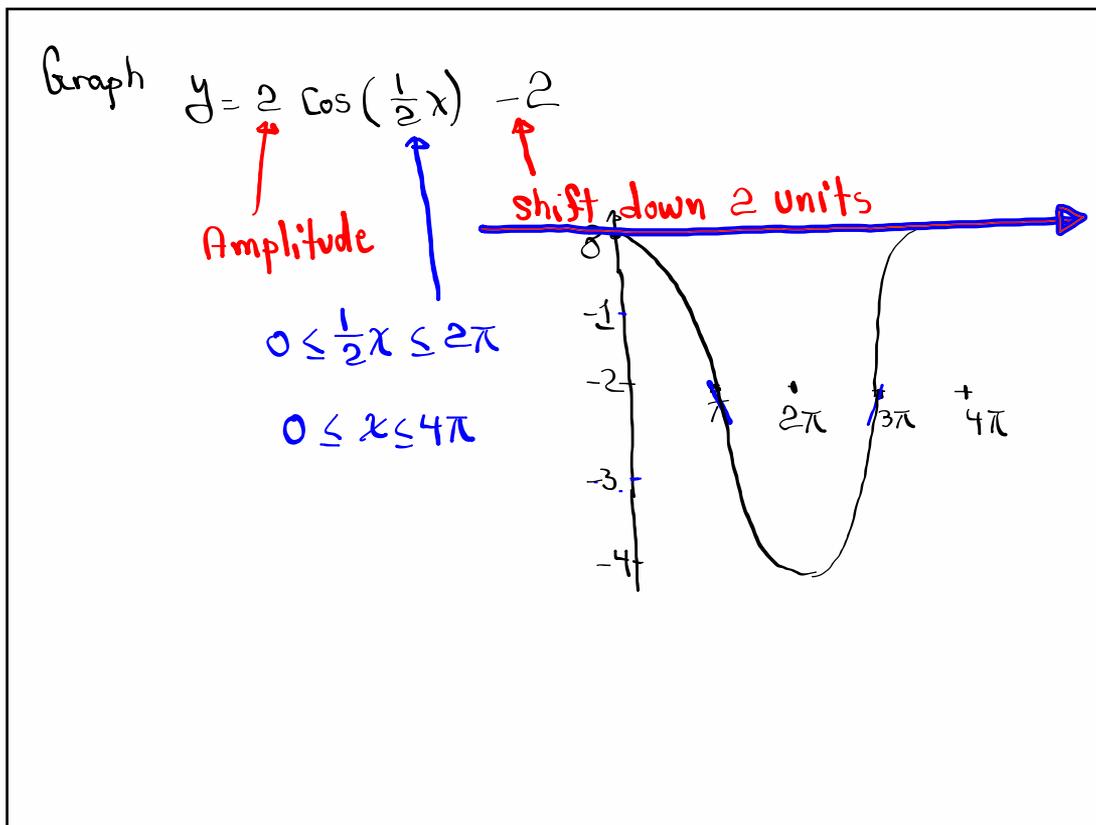
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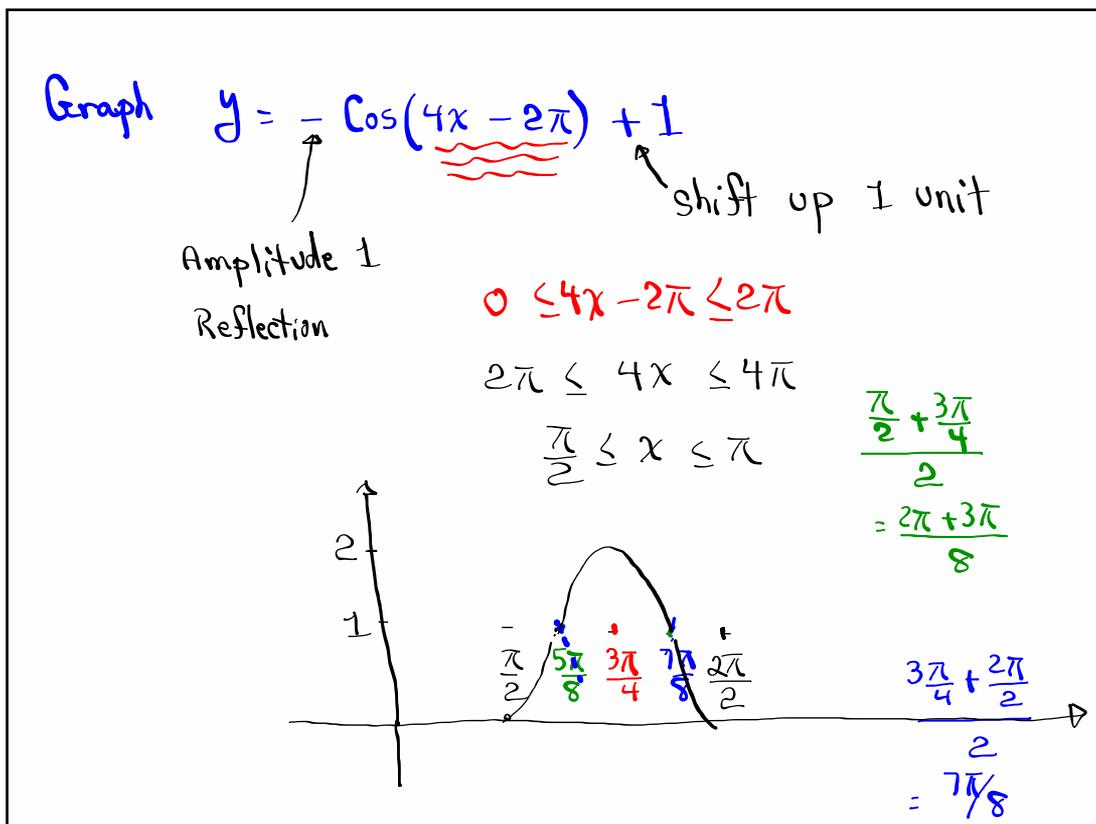
Jan 11-11:54 AM



Jan 11-11:56 AM



Jan 11-12:05 PM



Jan 11-12:11 PM